sustainability

MDPI

Article

Mathematical Model of the Plane-Parallel Movement of the Self-Propelled Root-Harvesting Machine

Volodymyr Bulgakov [1], Simone Pascuzzi [2,*], Francesco Santoro [2] and Alexandros Sotirios Anifantis [2]

[1] Department of Mechanics, Faculty of Construction and Design, National University of Life and Environmental Sciences of Ukraine, 03041 Kyiv, Ukraine; vbulgakov@meta.ua

[2] Department of Agricultural and Environmental Science, University of Bari Aldo Moro, Via Amendola 165/A, 70126 Bari, Italy; francesco.santoro@uniba.it (F.S.); alexandrossotirios.anifantis@uniba.it (A.S.A.)

* Correspondence: simone.pascuzzi@uniba.it; Tel./Fax: +39-0805442214

Received: 22 August 2018; Accepted: 8 October 2018; Published: 10 October 2018

check for updates

Abstract: The harvest techniques and the employed machines are important factors in reducing soil loss due to root crop harvesting. Furthermore, the deviation of the working organs of the self-propelled sugar root harvesting machines from the axis of the row also leads to significant losses and damage to sugar beetroots. Therefore, the self-propelled machine units must move in a horizontal plane with a high degree of accuracy. The purpose of this study is to increase the efficiency of the self-propelled harvester by analyzing its plane-parallel motion and evaluating its constructive and kinematic parameters. In order to determine the influence of these parameters on the plane-parallel motion of the self-propelled root harvesting machine, its mathematical model has been calculated. Furthermore, experimental tests were executed in order to evaluate the degree of damage to sugar beetroot crops during their digging, depending on the magnitude of the deviations of the center of the digging tool. The results of this trials highlighted that if the crop row deviates from the conventional axis line by 10 mm, the root crop damage exceeds is 21.7% and at deviations by 70 mm, the damage exceeds 67%. The theoretical study of the trajectory of the center of the outside digging tool and the experimental evaluation of its work (in terms of the quality of harvesting with deviations in its trajectory of motion) formally confirm the coincidence of all the studies—both theoretical and experimental. The use of the model of the plane-parallel movement of the self-propelled root harvesting machine then improves the quality parameters of the technological process.

Keywords: root-harvesting machine; plane-parallel motion; self-propelled machine; driving automaton

1. Introduction

A noteworthy soil erosion process is strictly linked to the soil loss due to root crop harvesting (SLCH) [1–4]. SCLH takes place whenever untied and fine soil and rock scraps stick to root crops such as sugar beet, chicory, carrot, potatoes, during their harvesting and dislodgement from the field. Furthermore, the SLCH also has environmental and economic outcomes produced by soil carriage, flushing of crop-roots, warehousing and disposal of the soil [5,6]. Many studies analyzed the elements affecting SLCH with the aim to find remedies and, among them, the prime suspects are the soil characteristics (texture, moisture, and/or organic matter content) [7–9].

The harvest techniques, the typology of the employed machines, and the adopted operative parameters (harvesting speed, harvesting depth, and so on) are also important factors in producing SLCH [10–12].

Other studies, with reference to self-propelled root harvesting machines that perform the harvesting process while moving along rows of beetroots and, generally, to all harvesting and other machine units that carry out the technological process by moving along fixed trajectories, pointed out that the qualitative performance of these machines is strictly determined by the nature of their plane-parallel motion [13–15]. Really, it has been emphasized that the main type of movement of self-propelled harvesting agricultural machines is their plane-parallel movement since this type of movement determines the agrotechnical and operational-technical indicators, as well as the productivity of the work [13–15].

Besides the problems linked to the production of SLCH, the deviation of the working organs of the self-propelled sugar root harvesting machines from the axis of the row by only 10–12 mm also leads to losses and damage to sugar beetroots, reaching more than 20% [16]. Therefore, it is necessary that self-propelled machine units move in a horizontal plane with a high degree of accuracy. Such motion is determined not only by the forward speed, but also by the design parameters of their running systems, by the type of excavating working bodies and their placement relative to the support wheels, by the use of different types of steering gears, the driving machine, and so on.

Several studies investigated the theory and practice connected to the employment of self-propelled units, giving important advantages to their use in working conditions [13–15]. At the same time, the theoretical models and the practical results developed and obtained till now do not allow us to evaluate the design and technological parameters of a self-propelled unit and, as a result, they do not provide an opportunity to achieve a significant increase of its technical and economic performance.

The search for a scientifically grounded solution aimed at eliminating these shortcomings was the basis of this study. The purpose of this study is then to increase the efficiency of the self-propelled harvester by analyzing its plane-parallel motion and evaluating its constructive and kinematic parameters. In order to determine the influence of these parameters on the plane-parallel motion of the self-propelled root harvesting machine, its mathematical model has been calculated. Furthermore, experimental tests were executed in order to evaluate the degree of damage caused to sugar beetroot crops during their digging, depending on the magnitude of the deviations of the center of the digging tool.

2. Materials and Methods

2.1. Theoretical Research

Theoretical research is based on the basic principles of theoretical mechanics, tractor theory, statistical dynamics, and the theory of automatic control of linear dynamical systems, which reproduce statistically random perturbing input influences, as well as methods for compiling programs and performing calculations on a PC [17–20].

2.1.1. Equivalent Scheme of a Self-Propelled Root Harvesting Machine

The first stage of the theoretical study was the construction of an equivalent scheme of the self-propelled root harvesting machine used for the experimental tests, whose main technical characteristics were the following: a power engine of 88 kW, a working width of 2.7 m (6 rows of root crops sown with 0.45 m inter-row spacing); a working speed in the 1.5 to 3.0 m s^{-1} range; and a performance in the 1.5 to 2.5 ha h^{-1} range (Figure 1).

The equivalent scheme of the aforesaid machine includes the elements of the motion system of the machine, the feeler wheels, and digging working bodies (Figure 2). The motion is considered relatively to the fixed Cartesian coordinates $OXYZ$, while the axes OX and OY form a horizontal plane of the surface of the field, and the axis OZ is directed vertically upwards. It is assumed that when moving along the surface of the field, all points of the running system and working organs of the root harvesting machine are moved only in planes parallel to the plane OXY. Forces that arise when considering such a plane-parallel motion, i.e., forces in the horizontal plane are represented in the form of concentrated forces having constant and maximum values. The moment of inertia of the self-propelled root harvesting machine has a constant value despite the fact that during the technological process, there is a continuous change and redistribution of masses inside (on cleaning working organs, the bunker of excavated root crops, etc.).

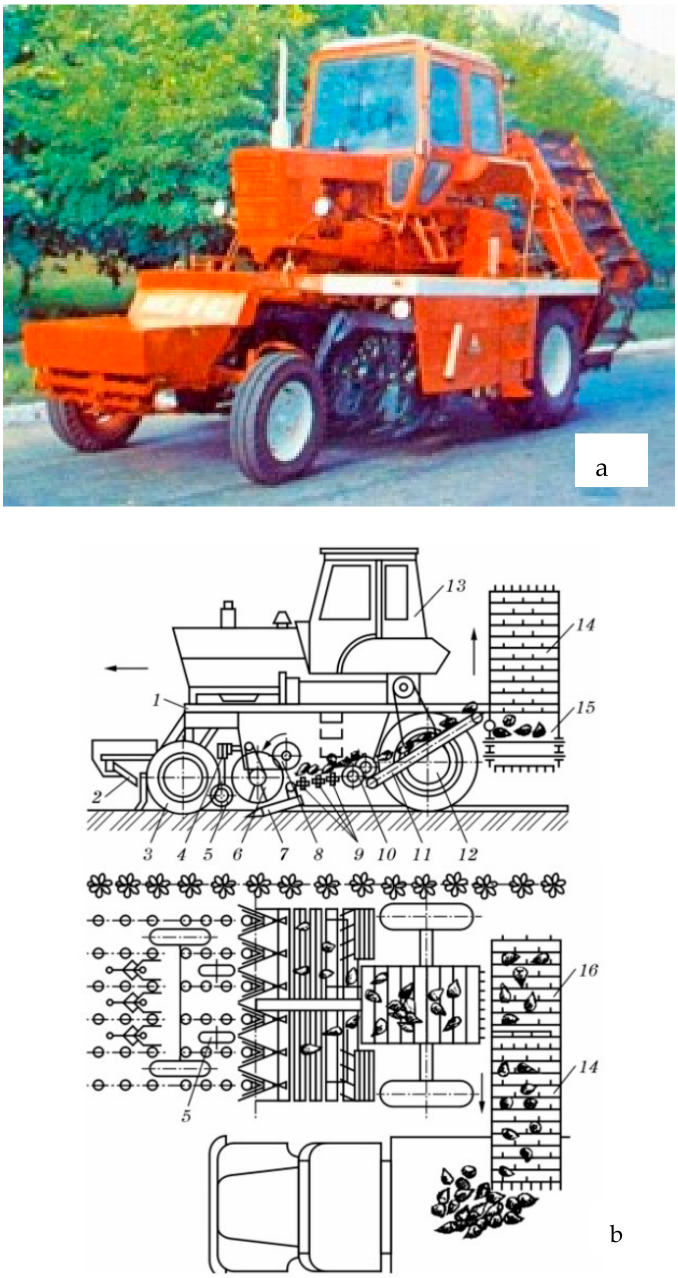

Figure 1. The self-propelled root harvesting machine. (**a**) view in the transport position; (**b**) working scheme: 1: frame, 2: driving machine in rows; 3: front steerable wheels; 4: digging section; 5: feeler wheels for digging sections; 6: root lifts; 7: excavated working bodies of rotary type; 8: lobed bitter-excavator of root crops; 9: receiving biter conveyor cleaner; 10: screw conveyor-cleaner; 11: longitudinal conveyor; 12: driving wheel; 13: power unit mounted on the frame; 14: discharge conveyor; 15: bunker-storage; 16: lower conveyor of bunker-storage.

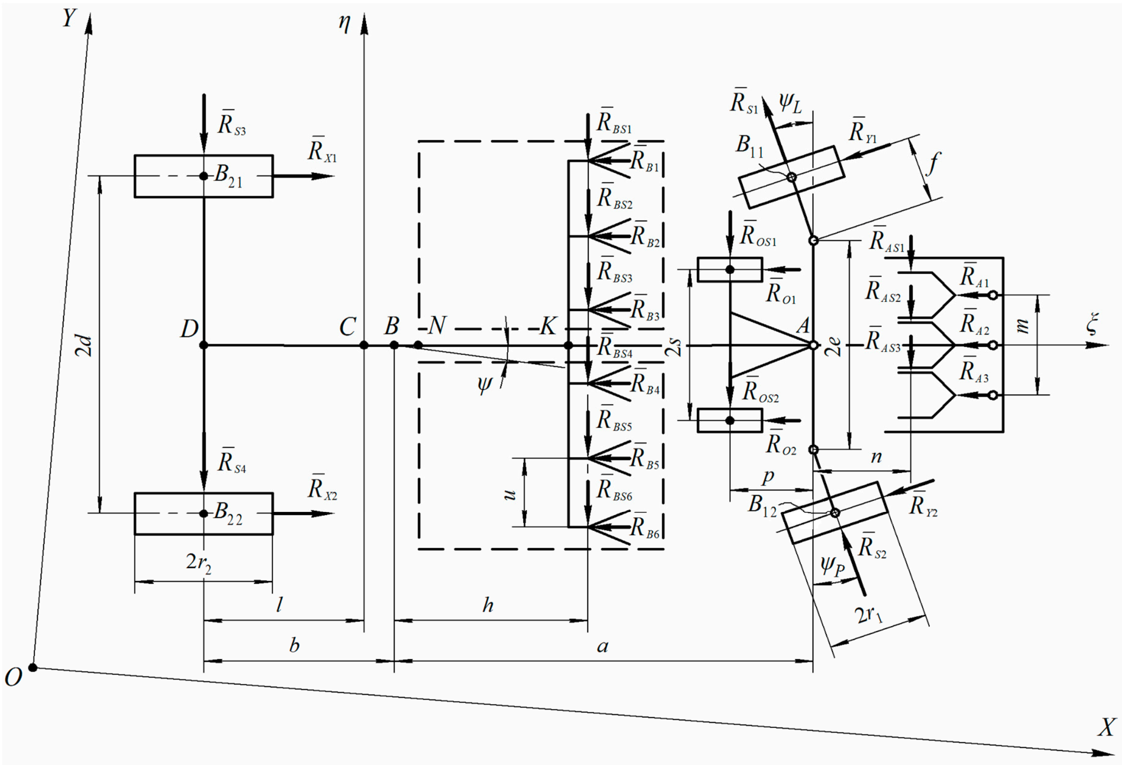

Figure 2. The equivalent scheme of the plane-parallel motion of the self-propelled root harvesting machine in the horizontal plane.

The mobile system of Cartesian coordinates $C\xi\eta\zeta$ is rigidly connected to the body of the machine, constantly connected with it. In this case, the axis $C\zeta$ is directed vertically upwards, the axis $C\xi$ coincides with the longitudinal axis of the machine, and the axis $C\eta$ is parallel to the axis of the driving wheels and is directed to the left of the direction of moving. The nature of the movement of the root harvesting machine in the plane under analysis is determined by the values of the soil reactions on the wheels of the motion and feeler systems, the total resistance on the excavating working organs, and also the resistances that arise on other working organs [21,22]. The position of the machine in the plane under consideration of OXY, in general, will be determined by the coordinates X and Y of point C and the heading angle Ψ, counted from the axis OX to the axis $C\xi$ [23]. The orientation of the axes of the left and right steered wheels relative to the body of the machine is determined by the angles Ψ_L and Ψ_P. These coordinates can be taken as generalized coordinates of the dynamical system under consideration.

2.1.2. Mathematical Model

For the derivation of differential equations for the plane-parallel motion of the root-harvesting machine, it is most convenient to use the equations of dynamics in the Lagrange form of the second kind [24,25]:

$$\frac{d}{dt}\cdot\frac{\partial T}{\partial \dot{q}_i} - \frac{\partial T}{\partial q_i} = Q_i \tag{1}$$

To determine the kinetic energy T of the system under consideration, it is necessary to establish the kinematic elements of the movement of the root-harvesting machine and, above all, the velocity of the reduced centers of mass of its body.

The values of the velocity of the center of mass of the root harvesting machine are obtained with the following equation [15]:

$$V_B^2 = \dot{X}^2 + \dot{Y}^2 + 2(b-l)\dot{\psi}\left(-\dot{X}sin\psi + \dot{Y}cos\psi\right) + (b-l)\dot{\psi}^2 \tag{2}$$

The kinetic energy of the entire root-harvesting machine will be

$$T = T_1 + T_2 \tag{3}$$

The components of the Equation (3) are determined by the following relationships:

$$T_1 = \frac{1}{2}MV_B^2 = \frac{1}{2}M\left[\dot{X}^2 + \dot{Y}^2 + 2(b-l)\dot{\psi}\left(-\dot{X}sin\psi + \dot{Y}cos\psi\right) + (b-l)\dot{\psi}^2\right] \tag{4}$$

$$T_2 = \frac{1}{2}J_M\dot{\psi}^2 \tag{5}$$

After substituting the values of Equations (4) and (5) into Equation (3), the kinetic energy of the entire root-harvesting machine is finally obtained:

$$T = \frac{1}{2}\left\{M\left[\dot{X}^2 + \dot{Y}^2 + 2(b-l)\dot{\psi}\left(-\dot{X}sin\psi + \dot{Y}cos\psi\right) + (b-l)\dot{\psi}^2\right] + J_M\dot{\psi}^2\right\} \tag{6}$$

By generalized coordinates X, Y, and ψ, generalized forces are defined. For this purpose, the equations for the elementary work of forces δW on the path of the possible displacements of the given system are used [22]:

$$\delta W = \sum_{i=1}^{n} Q_i\delta q_i \tag{7}$$

Thus, from the generalized coordinate X, the equation for the generalized force Q_x takes the form:

$$\begin{gathered} Q_X = \left(R_{X_1} + R_{X_2} - R_{O_1} - R_{O_2} - R_{A_1} - R_{A_2} - R_{A_3}\right)cos\psi + \left(R_{S_3} + R_{OS_1} + R_{OS_2} + R_{S_4}\right)sin\psi \\ -R_{Y_1}cos(\psi+\psi_L) - R_{S_1}sin(\psi+\psi_L) - R_{Y_2}cos(\psi+\psi_P) - R_{S_2}cos(\psi+\psi_P) \\ -\sum_{i=1}^{6} R_{B_i}cos\psi + \sum_{i=1}^{6} R_{BS_i}sin\psi \end{gathered} \tag{8}$$

From the generalized coordinate Y, the equation for the generalized force Q_y can be represented:

$$\begin{gathered} Q_y = R_{S_1}cos(\psi+\psi_L) - R_{Y_1}sin(\psi+\psi_L) - R_{S_2}cos(\psi+\psi_P) - R_{Y_2}sin(\psi+\psi_P) - \\ \left(R_{OS_1} + R_{OS_2} + R_{S_3} + R_{S_4}\right)cos\psi - \left(R_{A_1} - R_{A_2} - R_{A_3} + R_{O_1} + R_{O_2} - R_{X_1} - R_{X_2}\right)sin\psi - \\ \sum_{i=1}^{6} R_{BS_i}sin\psi - \sum_{i=1}^{6} R_{BS_i}cos\psi \end{gathered} \tag{9}$$

In the Equations (8) and (9), there are no projections of the lateral reactions of the soil on the feelers of the driving machine since the mechanism of their attachment to the frame allows their movement in the transverse plane.

From the generalized coordinate ψ, the generalized force Q_ψ has the dimension of the moment and can be determined by the following equation:

$$Q_\psi = M_{rot.} - M_{res.rot.} \tag{10}$$

where $M_{rot.}$ Is the moment of forces turning the car through an angle ψ and $M_{res.rot.}$ is the torque moment of resistance.

If you assume that the rotation of the machine during the movement at the time of the technological process of harvesting is carried out around the center of mass of the machine C, then, as follows from Figure 2, after substitution in Equation (10), we obtain the value of the generalized force Q_ψ [13]:

$$\begin{aligned} Q_\psi = R_{S_1}[cos\psi_L(a - fsin\psi_L) + sin\psi_L(e + fcos\psi_L)] + R_{S_2}[cos\psi_P(a + fsin\psi_P) - sin\psi_P(e + fcos\psi_P)] - \\ (R_{X_2} - R_{X_1})d - (R_{S_3} - R_{S_4})b - \sum_{i=1}^{6} R_{BS_i}h - \left(\sum_{i=1}^{3} R_{LB_i} - \sum_{i=1}^{3} R_{PB_i}\right) \\ 4.5u + (R_{OS_1} + R_{OS_2})(a - p) - (R_{O_1} + R_{O_2})s - (R_{A_1} + R_{A_2})m \end{aligned} \quad (11)$$

Substituting all the necessary quantities and carrying out the transformations provided by Equation (1), a system of differential equations for the plane-parallel motion of a self-propelled machine aggregate in expanded form is obtained:

$$\begin{cases} M\left[\ddot{X} - (b-l)\ddot{\psi}sin\psi + 2(b-l)\dot{\psi}^2cos\psi\right] = (R_{X_1} + R_{X_2} - R_{O_1} - R_{O_2} - R_{A_1} - R_{A_2} - R_{A_3})cos\psi + (R_{S_3} + R_{OS_1} + R_{OS_2} + R_{S_4})sin\psi \\ \quad + R_{Y_1}cos(\psi + \psi_L) - (R_{S_1} + R_{S_2})sin(\psi + \psi_L) - R_{Y_2}cos(\psi + \psi_P) - \sum_{i=1}^{6} R_{B_i}cos\psi + \sum_{i=1}^{6} R_{BS_i}sin\psi \\ M\left[\ddot{Y} - 2(b-l)\ddot{\psi} - 2(b-l)\dot{\psi}sin\psi\right] = (-R_{OS_1} - R_{OS_2} + R_{S_3} - R_{S_4})cos\psi + (-R_{A_1} - R_{A_2} - R_{A_3} - R_{O_1} - R_{O_2} + R_{X_1} + R_{X_2})sin\psi \\ \quad + R_{S_1}cos(\psi + \psi_L) - R_{Y_1}sin(\psi + \psi_L) + R_{S_2}cos(\psi + \psi_P) - R_{Y_2}sin(\psi + \psi_P) - \sum_{i=1}^{6} R_{B_i}sin\psi + \sum_{i=1}^{6} R_{BS_i}cos\psi \\ I_M\ddot{\psi} = R_{S_1}[cos\psi_L(a - fsin\psi_L) + sin\psi_L(e + fcos\psi_L)] + R_{S_2}[cos\psi_P(a + fsin\psi_P) - sin\psi_P(e + fcos\psi_P)] - (R_{X_2} - R_{X_1})d \\ \quad - (R_{S_3} - R_{S_4})b + \sum_{i=1}^{6} R_{BS_i}h - \left(\sum_{i=1}^{3} R_{LB_i} - \sum_{i=1}^{3} R_{PB_i}\right)4.5u + (R_{OS_1} + R_{OS_2})(a - p) - (R_{O_1} + R_{O_2})s - (R_{A_1} + R_{A_2})m \end{cases} \quad (12)$$

To solve the system of differential Equation (12) that exactly simulates the plane-parallel motion of a self-propelled root harvesting machine, a harmonic function that describes the amplitude and frequency of the oscillations of the deviations of the conventional row of sugar beetroot crops on the surface of a beetroot field has been used. Really, it is quite obvious that different causes affect the growth and development of sugar beetroots in the soil, as a result, causing the linearity of their rows to be disturbed. Deviations are also possible under the influence of tools used in the care of crops. Therefore, even the conditionally rectilinear movement of the self-propelled root harvesting machine is actually carried out with the harmonic oscillations of its center of mass in the horizontal plane, especially for the extreme excavating working organs. This kind of harmonic movement of the root harvesting machine may be caused by different reactions to the wheels of the root harvesting machine, different pressures in the pneumatic tires of its wheels, the variable hardness of the soil when the excavating tools move in it, and so on.

2.2. Experimental Tests

Field experimental studies were executed to determine the degree of damage to sugar beetroot crops during harvesting, depending on the magnitude of the deviations of the outside digging unit of the root crop machine from the conditional axial line of a row of their crops. The aforementioned self-propelled root harvesting machine carried out the harvesting of sugar beetroots, from which the tops were cut with high quality (according to the existing agrotechnological requirements) (Figure 1). The tests considered different amplitudes of displacement from the rectilinear movement of digging working organs, starting with a very small value and up to its maximum value, at which cleaning was no longer possible and the degree of damage to the excavated root crops was carefully measured.

3. Results and Discussion

3.1. Numerical Results

The numerical solution of the system (Equation (12)) was carried out on a PC by the Runge–Kutta–Fehlberg method of the 4th–5th orders [26]. Based on the results of the solutions, the values of the change in the generalized coordinates in time were obtained, which made it possible to create the trajectories of the motion of the center of mass of the root harvesting machine, as well as the center of the outside digging working body with a different value of the velocity (Figure 3). It can be seen from the presented graphs that the trajectories of the movement of the root harvesting machine have a form close to a harmonic, since, during the performance of the technological process of harvesting, the machine is under the influence of external perturbing influences constantly changing in magnitude and direction. At the initial moment of time, the trajectories of motion represent a smooth (uncontrolled) change in the curves to a certain value, then, when the steerable wheels are rotated (manually or by means of a driving machine), the lateral and normal reactions are redistributed, and the motion is reversed.

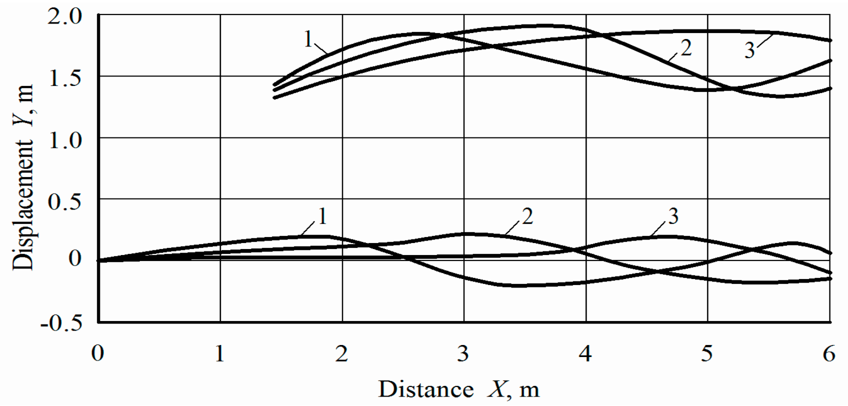

Figure 3. The trajectory of the motion of the center of mass of the root harvesting machine and the center of the outside digging working body at different velocities ($m{\cdot}s^{-1}$): 1 = 0.75; 2 = 1.68; 3 = 3.00.

The amplitude of such vibrational motions does not exceed 0.23 m. In the period of steady motion, the frequency of these oscillations increases to a certain value equal to 0.6 Hz. With a higher value of the speed of motion (Figure 3, curve 3), the deviation intervals from the rectilinear direction increase. Thus, the obtained system of differential equations (Equation (12)) contains constructive parameters of the root-harvesting machine. By changing these parameters, it is possible to achieve such values of kinematic parameters that will contribute to a more section movement of the self-propelled root crop machine in the horizontal plane. Further, the values of soil surface reactions on pneumatic wheels of running and feeler systems of a given machine unit can be specified using known dependencies from the theory of cars, tractors, and other earth-moving machines. This will also make it possible to optimize the parameters of the plane-parallel motion, and therefore, to improve the quality parameters of the technological process of the machine aggregate.

3.2. Experimental Results

Figure 4 reports the damage to sugar beetroot crops during harvesting by the root-picking machine due to the deviation of the outside digging working unit from the conditional root arrangement axis in a row. The damage to sugar beetroot crops was assessed by the size and magnitude of the trauma of the lateral parts of the root cones, cuts and chips of their heads, breaking off the tail sections, parts of the outer surfaces subjected to shocks and cuts.

The graphical dependence highlights that the degree of damage to the sugar beetroot crops, when harvested by a given root harvesting machine, has a form close to linear in the range of deviations up to 30 mm and then becomes close to the exponential in the 30 to 70 mm range, indicating a sharp increase in these injuries (Figure 4). Moreover, if the crop row deviates from the conventional axis line by 70 mm, the root crop damage exceeds 67%. A further increase in these deviations makes it virtually impossible to further harvest the sugar beetroots. The results of the trials point out the following average indicators: with deviations of the digging working organ from the conditional center line of the row of sugar beet crops up to 10 mm, the damage is 21.7% (including strongly damaged: 2.5%); with deviations up to 30 mm, the damage is 48.2% (including strongly: 8.2%); at deviations up to 60 mm and more, the damage is 67.5% (including strongly: 11.8%) (Figure 4).

Further experimental tests carried out with the same root-harvesting machine have underlined that the deviations of the longitudinal axes of excavated working bodies in the 10 mm to 70 mm range lead to total (irrecoverable) sugar beet yield losses within the 8.3 to 15.0% range.

Therefore, the experimental results emphasize that the deviations from the conditional axial line of the row of crops of the digging working organ of the root-harvesting machine significantly determine the quality of the harvested sugar beetroots. It is then necessary to ensure a steady plane-parallel movement of the machine and its excavating working organs in order to obtain the required quality of harvesting.

The numerical results reported in Figure 3 highlight that the level of stable movement is more favorable at the maximum possible forward speed (close to 3.0 m s^{-1}) for the given construction and power parameters of the root-harvesting machine. Thus, the theoretical study of the trajectory of the center of the outside digging tool and the experimental evaluation of its work (in terms of the quality of harvesting with deviations in its trajectory of motion) in the experimental part formally confirm the coincidence of all the studies—both theoretical and experimental. However, the steady movement in the horizontal plane is provided by the whole set of operational and technical conditions. So, the pressure in the tires of the front driven wheels should be in the set limits by the machine-maker and also the clearances in the steering must not exceed the set values. In connection with this, the root harvesting machine must have precise and strict adjustments (clearances in the mechanical and hydraulic servo-mechanisms should be absent), adjusted to respect the translational speed of the machine (close to 3.0 m s^{-1}) and by taking into account the characteristics of the conditional center lines of rows of crops, the average size of the upper parts of the root crops (the size and shape of the heads of root crops after cutting to the top of the foliage), and so on.

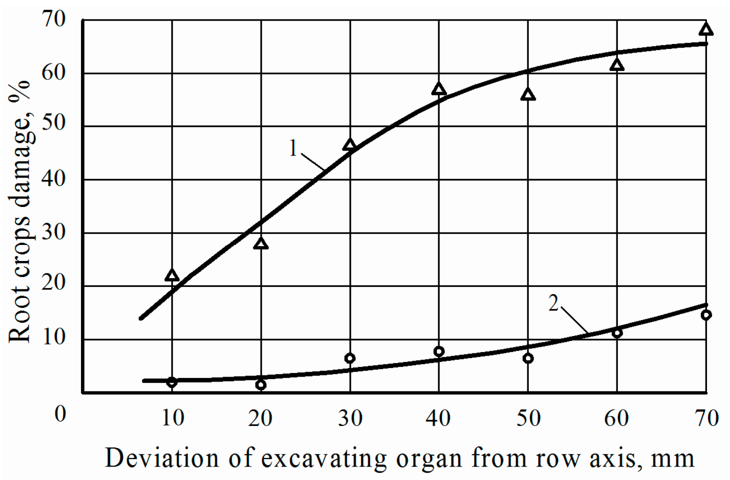

Figure 4. The dependence of the sugar beetroot crop damage from the magnitude of the deviation of the longitudinal axis of the outside digging unit from the conditional centerline of the row of crops: 1 = total damage; 2 = heavy damage.

4. Conclusions

1. It was determined that the dynamic system of the self-propelled harvesting aggregate can be considered as conservative with practically absent dissipative processes.
2. Since the self-propelled root harvester is a complex, multi-mass dynamic system, the method of compiling systems of differential equations in the Lagrange form of the second kind is most suitable for investigating the motion of such a system. The resulting system of second-degree differential equations is nonlinear, the solution of which is produced by numerical methods on the PC. The obtained system of differential equations contains constructive parameters of the root-harvesting machine. By changing these parameters, it is possible to achieve such values

that will contribute to a more stable movement of the self-propelled root crop machine in the horizontal plane.

3. The use of the obtained mathematical model makes it possible to optimize the parameters of the plane-parallel motion and to improve the quality parameters of the technological process.
4. Further analytical studies on this model can be focused on other solutions of this mathematical model of the PC, for cases when disturbing effects on the elements of a self-propelled root harvester can be represented in the form of statistical functions. The results obtained in the general form create prerequisites for the theoretical study of other mobile machine aggregates.

Author Contributions: V.B. conceived the paper; V.B., S.P., F.S. and A.S.A. performed the work and wrote the paper.

Funding: This research received no external funding.

Conflicts of Interest: The authors declare no conflict of interest.

Nomenclature

B	center of mass of the root harvesting machine
C	pole (a fixed point on the machine body located on its longitudinal axis)
A, D	midpoints of the interaxial distances between the front driven and rear drive wheels, respectively
B_{11}, B_{12}, B_{21}, B_{22}	centers of mass points of the front driven and rear drive wheels
N	intersection point of the suspension axis of the digging sections and the longitudinal axis of the machine
A	hinge point of the frame of the feeler wheels
$AB = a$	distance from the center of mass B to the front axle
$DB = b$	distance from the center of mass B to the rear axle
$DC = l$	distance from pole C to the rear axle
$BK = h$	distance from pole C to the axis of the suspension of the digging sections
p	distance from the front axle to the axis of the feeler wheels
n	distance from the front axle to the axis of the feelers
m	distance between the suspension points of the outside feelers
$2e$	distance between the axis of turns of steerable wheels
u	distance between the axis of symmetry of neighboring excavating working bodies
f	length of the swivel pin of the steering gear
$2r_1$ and $2r_2$	diameters of the steerable and driving wheels, respectively
$2d$ and $2s$	width of the track of the driving and feeler wheels
$AD = a + b$	longitudinal base of root harvesting machine
J_M	moment of inertia of the root harvesting machine relative to the vertical axis $C\xi$
M	mass of the root harvesting machine
$M_{res.rot.}$	torque moment of resistance
$M_{rot.}$	moment of forces turning the root harvesting machine through an angle ψ
q_i	generalized coordinates
Q_i	generalized forces
R_{y1}, R_{y2}	soil reactions acting on the steerable wheels
R_{x1}, R_{x2}	soil reactions acting on the driving wheels
R_{O1}, R_{O2}	soil reactions acting on the feeler wheels
R_{A1}, R_{A2}, R_{A3}	soil reactions acting on the feelers
$R_{B1}, R_{B2}, \ldots, R_{B6}$	total resistance reactions during the movement of excavated working organs in the soil
R_{S1}, R_{S2}, R_{S3}, R_{S4}	lateral reactions of the soil, acting on the steerable and driving wheels of the root-harvesting machine when moving along the axis Y
R_{OS1}, R_{OS2}	lateral reactions of the soil, acting on the feeler wheels of the root-harvesting machine when moving along the axis Y

$R_{BS1}, R_{BS2}, \ldots, R_{BS6}$	lateral reactions of the soil, acting on the digging working organs
$R_{AS1}, R_{AS2}, R_{AS3}$	lateral reactions of the soil, acting on the sensors of the feelers
T	energy of the system
T_1 and T_2	kinetic energy, respectively, of the translational motion of the root harvesting machine and its rotation around the pole C
V_B	velocity of the center of mass
ψ	heading angle
ψ_L and ψ_P	angles to determine the orientation of the axes of the left and right steered wheels relative to the body of the machine
$\dot{\psi}$	angular speed of rotation of the machine around the pole C
δq_i	variations of the corresponding generalized coordinates

References

1. Oshunsanya, S.O.; Yu, H.; Li, Y. Soil loss due to root crop harvesting increases with tillage operations. *Soil Tillage Res.* **2018**, *181*, 93–101. [CrossRef]
2. Parlak, M.; Cicek, G.; Blano-Canqui, H. Celery harvesting causes losses of soil: A case study in Turkey. *Soil Tillage Res.* **2018**, *180*, 204–209. [CrossRef]
3. Faraji, M.; Chakan, A.A.; Jafarizadeh, M.; Behbahani, A.M. Soil and nutrient losses due to root crops harvesting: A case study from Southwestern Iran. *Arch. Agron. Soil Sci.* **2017**, *63*, 1523–1534. [CrossRef]
4. Dada, P.O.O.; Adeyanju, O.R.; Adeosun, O.J.; Adewumi, J.K. Effects of soil physical properties on soil loss due to manual yam harvesting under a sandy loam environment. *Int. Soil Water Conserv. Res.* **2016**, *4*, 121–125. [CrossRef]
5. Yu, H.Q.; Li, Y.; Zhou, N.; Chappell, A.; Li, X.Y.; Poesen, J. Soil nutrient loss due to tuber crop harvesting and its environment impact in the North China Plain. *J. Integr. Agric.* **2016**, *15*, 1612–1624. [CrossRef]
6. Oztas, T.; Ozbek, A.K.; Turan, M. The cost of soil lost from fields due to removal on harvested sugarbeet: A case study in Turkey. *Soil Use Manag.* **2002**, *18*, 236–237. [CrossRef]
7. Parlak, M.; Palta, C.; Yokus, S.; Blanco-Canqui, H.; Carkaci, D.A. Soil losses due to carrot harvesting in south central Turkey. *Catena* **2016**, *140*, 24–30. [CrossRef]
8. Pascuzzi, S.; Santoro, F. Analysis of the almond harvesting and hulling mechanization process: A case study. *Agriculture (Switzerland)* **2017**, *7*, 100. [CrossRef]
9. Munoz-Romero, V.; Benitez-Vega, J.; Lopez-Bellido, R.; Fontan, J.; Lopez-Bellido, L. Effect of tillage system on the root growth of spring wheat. *Plant Soil* **2010**, *326*, 97–107. [CrossRef]
10. Yeboah, S.; Lamptey, S.; Zhang, R.; Li, L.L. Conservation tillage practices optimize root distribution and straw yield of spring wheat and field pea in dry areas. *J. Agric. Sci.* **2017**, *9*, 37–47. [CrossRef]
11. Schneider, F.; Don, A.; Hennings, I.; Schmittmann, O.; Seidel, J.S.J. The effect of deep tillage on crop yield —What do we really know? *Soil Tillage Res.* **2017**, *174*, 193–204. [CrossRef]
12. Oshunsanya, S.O. Alternative method of reducing soil loss due to harvesting of sweet potato in Nigeria: A case study of low input agriculture. *Soil Tillage Res.* **2016**, *158*, 49–56. [CrossRef]
13. Smirnov, G.A. *Theory of Movement of Wheeled Vehicles*; Mechanical Engineering: Moscow, Russia, 1990.
14. Wong, J.Y. *Theory of Ground Vehicles*, 3rd ed.; Wong, J.Y., Ed.; John Wiley & Sons Inc.: Hoboken, NJ, USA, 2001.
15. Vasilenko, P.M. *About the Technique of Mechanical and Mathematical Research in the Development of Problems of Agricultural Machinery*; BTI GOSNITI: Moscow, Russia, 1962.
16. Zuev, N.M. Placement of root crops in rows and quality of harvesting. *Sugar Beet* **1980**, *10*, 24–29.
17. Dreizler, R.M.; Ludde, C.S. *Theoretical Mechanics*; Springer: Berlin, Germany, 2010. [CrossRef]
18. Bhatia, N.P.; Szegő, G.P. *Stability Theory of Dynamical Systems*; Springer: Berlin, Germany, 2002; ISBN 978-3-540-42748-3.
19. Wu, D.H. A theoretical study of the yaw/roll motions of a multiple steering articulated vehicle. *Proc. Inst. Mech. Eng. Part D J. Autom. Eng.* **2001**, *215*, 1257–1265. [CrossRef]
20. Anifantis, A.S.; Colantoni, A.; Pascuzzi, S. Thermal energy assessment of a small scale photovoltaic, hydrogen and geothermal stand-alone system for greenhouse heating. *Renew. Energy* **2017**, *103*, 115–127. [CrossRef]
21. Vanessa, M.D.; Johannes, A.P.; Andrea, S.; Loic, P.; Mathieu, J.; Lianhai, W. Modeling root-soil interactions using three-dimensional models of root growth, architecture and function. *Plant Soil* **2013**, *372*, 93–124.

22. Vasilenko, P.M.; Vasilenko, V.P. *Method for Constructing Computational Models for the Operation of Mechanical Systems (Machines and Machine Aggregates)*; USHA: Kiev, Ukraine, 1980.
23. Lurie, A.I. *Analytical Mechanics*; Springer: Berlin/Heidelberg, Germany, 2002; ISBN 978-3-540-42982-1.
24. Bulgakov, V.; Pascuzzi, S.; Ivanovs, S.; Kaletnik, G.; Yanovich, V. Angular oscillation model to predict the performance of a vibratory ball mill for the fine grinding of grain. *Biosyst. Eng.* **2018**, *171*, 155–164. [CrossRef]
25. Pascuzzi, S.; Anifantis, A.S.; Blanco, I.; Scarascia Mugnozza, G. Electrolyzer performance analysis of an integrated hydrogen power system for greenhouse heating a case study. *Sustainability* **2016**, *8*, 629. [CrossRef]
26. Forsythe, G.F.; Malcolm, M.A.; Moler, C.B. *Computer Methods for Mathematical Calculations*; Prentice Hall Professional Technical Reference; Prentice Hall: Upper Saddle River, NJ, USA, 1977; ISBN 0131653326.