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Search algorithm for minimum reliability index of earth slopes[a]

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The method proposed is very interesting and operative for engineering estimations of sliding risk in multi-strata slopes and earth dams. However, the writers of this discussion have proposed a new method at the International Conference on Risk Analysis in Valencia (Cherubini et Al., 1998) which involves such a complementary aspect to that proposed by the Authors. It develops a graphical method for the homogeneous slopes to evaluate the reliability index β and compare its efficiency to the commonly used Safety Factor. This method is briefly discussed below.

The approach starts from Sparks' idea (1996) of constructing a characteristic limit state line (based on the deterministic condition of imminent failure in the c', ϕ' plane) for geometrically definite slopes. This line is drawn by means of expressions (Bishop' for example) where the "Safety Factor" is matched to 1.

Thus, c and ϕ values, determined as above, are plotted in a diagram "c' versus tanϕ'" or "c' versus ϕ'". It is easy to find an about linear relationship as c' and ϕ' varies within likely ranges of values. In the same diagram dots which represent the in situ slope state can be plotted. The "Safety Factor" of the slope considered is then assumed to be the ratio between two distances in the diagram: the distance between the origin and the point of the in situ state times the part of the previous distance which relies under the characteristic limit state line.

This deterministic point of view with its ambiguous "Safety Factor" has been overcome by Hasofer & Lind (1979) reliability index. They have proposed a quadratic, elliptical formulation of the reliability index (β_{HL}) reviewed by Low (1997) in a graphical perspective .

Looking from this graphical point of view (developed in the original space of random variables) if a one-standard-deviation (1-σ) dispersion ellipse has been defined, it can be seen that:

1. Each axis of the ellipse is parallel to a corresponding co-ordinate axis if the variables are not correlated; the

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dispersion ellipse is tilted when there is correlation;

2. Plotting the 1-σ ellipse, the β-σ ellipse and the failure surface together, we can see that, citing Low (1997): "to find the smallest ellipse that is tangent to the failure surface is then equivalent to find the most likely failure point". This is also consistent with Shinozuka (1983) who said that "the design point x* is the point of maximum likelihood if x is Gaussian, whether or not its components are correlated".

From these suggestions an Overall Approach as a general tool in the evaluation of the stability of the slope has been developed. The idea consists of linking the characteristic limit state line with the dispersion ellipse in order to graphically and numerically calculate the β_{min} and the β_{HL} of the slope.

Moreover, the characteristic resistance line corresponding to the c and ϕ limit values and the failure envelope according to the Casagrande's method (1950) can be plotted. When both the envelope and the line have been drawn on the stress plane we can graphically understand how far is the deterministic Safety Factor from the safety value when correlation coefficient and standard deviation of c' and ϕ' change.

In our opinion, this perspective enriches reliability theory concerning two aspects:

1. it makes the use of the reliability index easier and more direct;
2. it figures contemporarily information about mechanical parameters and stress values that are both important in designing processes.

Cases studied

The case proposed is very simple in geometrical characterisation in order to focus the attention on the meaning of the Overall Approach in geotechnical designing. The homogeneous slope analysed is sketched in Fig. A.

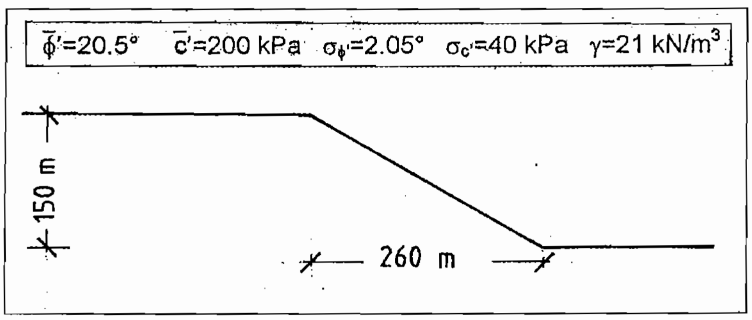

Figure A. Slope geometry and geotechnical values

The (c',ϕ') couples for FS = 1 have been calculated by a computer code based on Bishop's stability approach. This calculation has been done for all the circles centred in a wide grid of circle centres and for five circle toes. At first, a Characteristic Limit State Curve (CLSC) has been plotted. So that the curve defines two regions (the "Safe Region" and the "Unsafe Region") in the c'-ϕ' plane.

The CLSC is used as a failure curve in Low's probability approach (1997). Indeed, cohesion and friction angle has been considered random variables and represented in the random variables plane. Subsequently, onto the same plane Low's dispersion ellipses can be plotted.

It's easy to understand that if the ellipse 1-σ is all in the "Safe Region" the calculated β_{min} index will be certainly greater than the unit.

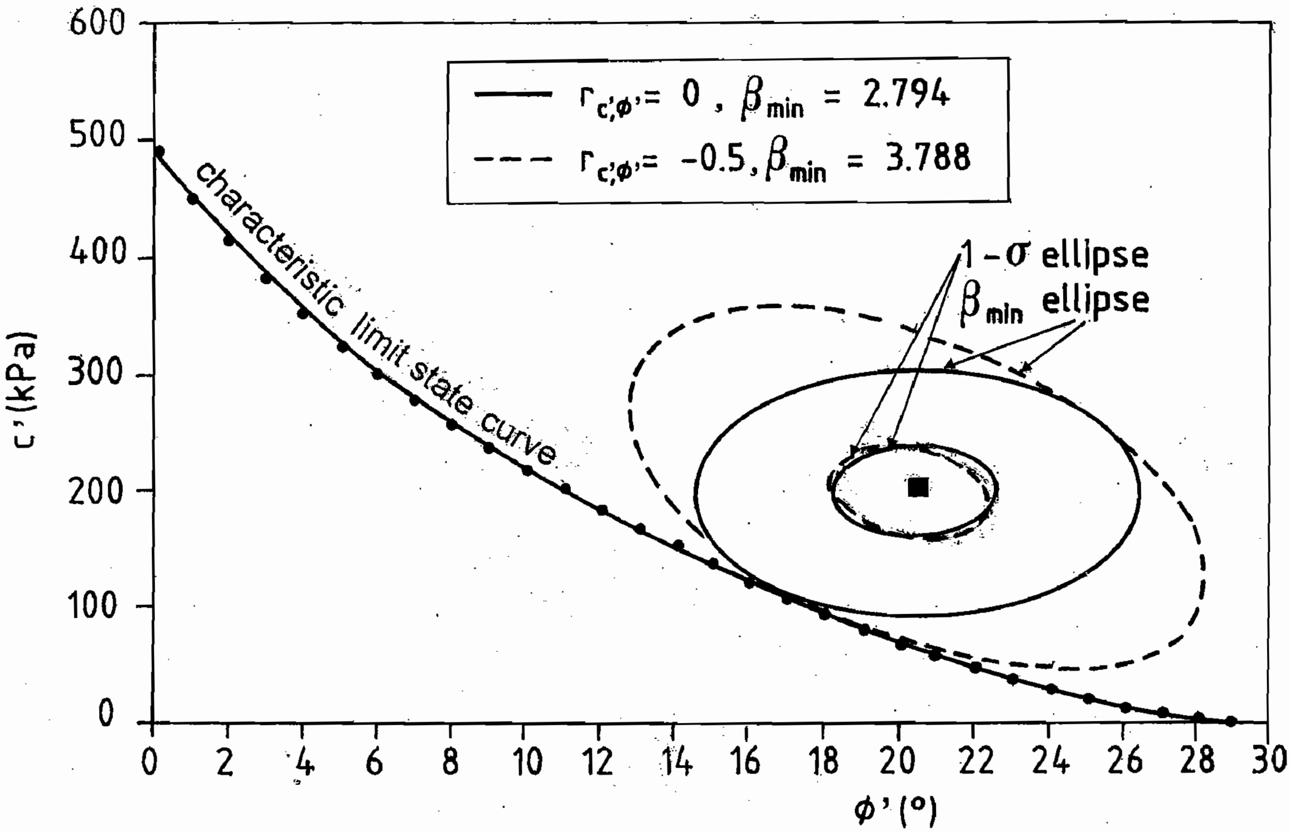

Figure B. Characteristic Limit State Curve and dispersion ellipses

Then, considering the point that Shinozuka defined as the "design point" (the point in which the ellipse is tangent to the CLSC) by Mohr-Coulomb criterion the "failure envelope" can be constructed. It will be predictable that in stress plane "failure envelope" will be tangent to the "characteristic resistance line". At the same time we can infer the value of "Safety Factor" through the comparison with Reliability Index value.

Table A. Summary of data and results

Mechanical properties of soil		
Mean value of ϕ' (°)		20.5
Mean value of c' (kPa)		200.0
Standard deviation of ϕ' (°)		2.05
Standard deviation of c' (kPa)		40.00
Coefficient of correlation between c' and ϕ'	0.00	-0.5
β_{min}	2.794	3.788
ϕ' value for β_{min} (°)	17.3	19.5
c' value for β_{min} (kPa)	107.3	80.1
ϕ' value for slip circle nearest to that of β_{min} (°)	17.0	19.0
c' value for slip circle nearest to that of β_{min} (kPa)	111.3	85.7
Δx of slip circle toe nearest to that of β_{min} from slope foot (m)	0.00	0.00
X of centre of slip circle nearest to that of β_{min} (m)	1040.0	1060.0
Y of centre of slip circle nearest to that of β_{min} (m)	330.0	360.0
Evaluation of Deterministic "Safety Factor"		
Deterministic "Safety Factor" F		1.471
σ value corresponding to F (kPa)		571.4
τ value on the "failure envelope" corresponding to F (kPa)		413.6
τ value on Characteristic Reliability Envelope corresponding to F (kPa)		281.3

Figure B shows that, for the same CLSC on c'-ϕ' plane, the change of correlation coefficient between cohesion and friction angle deals with different β_{min} values.

In the particular case of the absence of correlation between these two parameters ($r_{c',\phi'} = 0.00$), β_{min} is equal to 2.794 and the tangent point coordinates are c'=107.3 kPa and ϕ'=17.3°. On the other hand, for a negative correlation between the above mentioned parameters ($r_{c',\phi'} = -0.50$), β_{min} is equal to 3.788 and the tangent point coordinates are c'=80.1 kPa and ϕ'=19.5° (results in the two cases are summarised in Table A).

The two mentioned examples show that, when negative correlation between cohesion and friction angle exists, the tangent point between ellipse and CLSC is on the right of that determined in the absence of correlation.

The deterministic "Safety Factor" is calculated following Casagrande's definition as the ratio between available shear strength and required shear strength. Its value is insensitive to coefficient of correlation so that it remains constant (F=1.471) without taking into any account the β_{min} changed value.

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