Transactions on Ecology and the Environment vol 24, © 1998 WIT Press, www.witpress.com, ISSN 1743-3541

Reliability analysis of slopes

C. Cherubini, F. Santoro & G. Vessia
Institute of Applied Geology and Geotechnics,
Technical University, Via Orabona 4, 70125 Bari, Italy
Email: c.cherubini@area.ba.cnr.it

Abstract

In this paper has been described a new procedure by which the characteristic limit state line of a geometrically known slope can be calculated. This approach is based on an intuition, applied to a deterministic evaluation of slope stability, to look for a slope characteristic "failure curve". Subsequently, concerning probability perspective, has been considered Low's interpreting the reliability index. Linking prior studies together and to the Characteristic Resistance Envelope Procedure an Overall Approach has been developed. A simple computed case is included in order to show computing potentialities of this new approach.

1 Introduction

Problems concerning slope stability analysis are one of the most explored fields in geotechnical studies because of their influence on daily life. In some countries, for many years, the only answer to design and verify geotechnical structures, was the "Safety Factor" Approach which is until in wide use. This approach is based on a quickly evaluation of a ratio between soil strength and acting forces or moments. Indeed, it is obtained by different expressions at the numerator and at the denominator according to consider the same loads as resisting and acting terms in equilibrium conditions evaluation. Furthermore, laboratory or in situ tests provide soil mechanical properties which are affected by uncertainties that aren't usually taken in account.

However, Tabba[11] has found that design decisions regarding slope stability are relatively insensitive to the variability of geotechnical data when either the conventional or central "Safety Factor" is below 1.2 or above 1.8. But the variability of the input data has a big effect when the "Safety Factor" is between 1.2 and 1.8 which almost comprises the overall geotechnical "Safety Factor" range of use.

In order not to ignore mechanical properties variability and make geotechnical developments to support a more correct prediction of soil behaviours, Reliability Approach has been developed.

It starts admitting the possibility of failure for human structures and suggests that technical choices can be taken on the basis of a desirable probability of failure. During these years, firstly Cornell (1969) secondly Veneziano (1974), Hasofer & Lind[6], Vanmarcke (1977) and Low[9] (among others) have meant their efforts to both design and theoretically supported tool.

According to these works, our paper tries to provide an approach to make geotechnical engineers more confident in probability approaches to "Safety".

2 Variability of soil properties

Many measures of variability of physical - mechanical soil properties are available in literature. Those surveys have shown that intrinsic variability is significant and more often, mostly high even if errors in measures have a large part in the overall variability value.

In order to point out the most pregnant results, obtained up to now, some data referring to the coefficients of variation for certain geotechnical parameters are presented.

Numerous samples have been considered for which sufficient determinations have been carried out to calculate the coefficient of variations (Cherubini[4]). From those analyses has been drawn that:

1 - the mean value of the coefficient of variation for unit weight is about 6% and its minimum and maximum rounded values are respectively 1% and 28%.
2 - the mean value of the coefficient of variation for drained cohesion can be assumed as 33% with minimum value 13% and maximum 70%.

3 - for friction angle, the mean value of coefficient of variation is about 17% with minimum value 1% and maximum value 87%.

Plotting all these maximum and minimum values in the same diagram it should be noticed that 87% value is largely distant from corresponding mean values, while for ϕ >25° coefficient of variation are never greater than 20%. In some cases can be seen that for ϕ >30° the CVϕ (coefficient of variation) is always less than about 13÷14%.

Since this paper deals with cohesive and frictional soils, further aspect must be put in evidence: the presence of correlation between c and ϕ. In particular from literature (Cherubini[4]) negative values of $r_{c,\phi}$ have been reported. Harr (1987) also shows data concerning the values of correlation coefficient between c and ϕ. They are also negative in drained tests, varying from -0.24 to -0.70. Cherubini et al. (1990) have found consistent results for the Matera Blue Clays.

3 Resistance Envelope Procedure (REP)

This method, introduced as a principle in 1950 by Casagrande[1], then developed by Janbu[7] and Varghese[12], is a numerical method by which a deterministic slope "Safety Factor" as the ratio between actual shear strength of soil mass and mobilised shear strength can be calculated.

This procedure theoretically belongs to the equilibrium limit methods because of its starting assumptions:

1 - a constant "Safety Factor" along potential slide lines;

2 - assuming the rigid-plastic mechanism to simulate soil behaviour.

In this way, differences among the "Safety Factors" obtained from different failure criteria can be appreciated by calculating them just once. Geometrical and loading information is generally sufficient for the evaluation of average normal (σ_m) and tangential (τ_m) stresses which depend only on slope geometry and applied forces (as self weight, seismic actions, external loads).

So that, average stresses referring to potential failure slide lines can be used.

It is possible for Bishop's method, because it uses a circular failure line; for other methods average stresses can be referred to the conjunction line of the two extreme points (at the upstream and the downstream of the slope considered) of failure line.

Janbu[7] suggested assuming failure circular lines passing through slope toe with their centres lying onto the same vertical axis.

Then, all the couples (σ_m, τ_m) which represent stresses related to all potential failure slide lines can be plotted in Mohr's diagram; the most external point joining curve of Mohr's circles will be the resistance envelope procedure. The "Safety Factor" is then calculated as the ratio of two distances, measured along a vertical axe from the σ-axe, between failure line and resistance envelope where they are the closest. This ratio can be written: τ_f / τ where the first term is measured onto failure line and the second onto the resistance envelope. So far, it is evident that flexibility of the resistance envelope procedure is a powerful tool to evaluate slopes stability in a deterministic approach.

Instead, considering a reliability point of view, the foregoing explained procedure has two limits:

1 - it can be only used for homogeneous soil mass because it cannot consider various failure criteria (with different ϕ *and* c) for each soil strata. In these cases some simpler models can be attempted although approximated;
2 - it has developed for deterministic evaluations confining this method in a narrow contest.

4 Characteristic Limit State Line

In order to point out the handiness and the power of the resistance envelope method it has been applied in reliability probabilistic approach. Before explaining this new procedure, it is necessary to recall the works of two researchers in two different fields.

As a matter of fact, Sparks[10] trying to figure out the commonest students' misunderstandings in studying geotechnics, proposed a useful method to construct a characteristic limit state line of a slope for which all geometric parameters were known.

He suggested to put F = 1 corresponding to the limit state.

It was easy to find an about linear relationship between either "c versus tan ϕ" or "c versus ϕ" plotting these values in a diagram. Indeed, in the same diagram, he represented the in situ state of the natural slope which was given by characteristic values of c and ϕ. So that, for every failure lines only one characteristic limit state curve can be drawn.

Then, for determining "Safety Factor", related to the slope previously considered, he defined the ratio between two distances from the origin: the line of the in situ state times the part of previous line under the characteristic limit state line.

Sparks[10] showed the advantages of this method and we resume them here below:

1 - usage of c and tanϕ values is independent from the knowledge of shear strength; so, giving a fixed value to one of them the other one can be attained for F = 1;
2 - information about stresses and mechanical properties can be plotted together in order to link directly experimental stresses information to mechanical properties which can always easily be drawn.

On the other hand, during these years the reliability theory has been rapidly developed consistently with the awareness of variability of designing parameters in engineering projects.

Consequently, reliability approach has managed characteristic structural and geotechnical parameters as random variables. Commonly their probabilistic distributions have been assumed to be normal but many devices has also been provided to overcome this assumption or reducing different distributions to the normal one.

So that, in the reliability theory has been substituted “Safety Factor” by “Reliability Index” with also a deep change in its meaning. As a matter of fact, unless “Safety Factor” disadvantages were pointed out, the necessity of a handling but concerning a probabilistic nature design tool was the real difficulty for a wide application of Reliability theory.

Up to that time, in contrast of the ambiguity in attaining the deterministic "Safety Factor", Hasofer & Lind proposed a precise reliability index (1979) formulation as:

$$\beta_{HL} = \min_{x \in F} \sqrt{(x-m)^T C^{-1} (x-m)} \qquad (1)$$

where x is a vector representing the set of random variables; m is their mean value; C is the covariance matrix and F is the failure region separated by limit state surface from the safe region.

Low[9] has then reviewed this mathematical expression in a graphical way. He says that a more efficient interpretation of β is possible noting that eqn. (1) can be calculated by minimising that quadratic form. It's easy to understand as the mathematical formulation of an ellipse for twodimensional case subjected to the constrain that it just touches the surface of the failure region F or in other words the limit state surface.

Looking from the alternative point of view of Hasofer-Lind Index (in the original space of random variables) if a one-standard-deviation (1-σ) dispersion ellipsoid has been defined it can be seen that:

1 - each axis of the ellipsoid is parallel to a corresponding co-ordinate axis if the variables are not correlated; the dispersion ellipsoid is tilted when there is correlation;
2 - plotting the 1-σ ellipse, the β-σ ellipse and the failure surface together, you can see that, citing Low[9]: "to find the smallest ellipse that is tangent to the failure surface is then equivalent to finding the most probable failure point". This is also consistent with Shinozuka (1983) who said that "the design point x* is the point of maximum likelihood if x is Gaussian, whether or not its components are not correlated".

So that, using Sparks's intuition in applying to Low's perspective we have found a general way of evaluating the stability of a geometrical defined slope. This new approach, as the example proposed below will clearly show, enriches reliability theory in two ways:

1 - it makes reliability index to be used in such an easier and more direct way;
2 - it figures contemporarily information about mechanical parameters and stress values that are both important to display in designing processes.

5 Illustrative example

The application developed is very simple in geometrical characterisation for focusing attention on the meaning of an Overall Approach in geotechnical designing. The slope analysed is constituted of homogeneous soil whose geometry is sketched below:

A computer code has been written to calculate, using the Bishop's method with "Safety Factor" F=1, the (c,ϕ) couples.

This calculation has been done for all the circles centred in a wide grid of circle centres and for five circles toes (Fig.1).

In other words as a first step a *Characteristic Limit State Curve (CLSC)* has been plotted. In the previous definition the terms used have the following meanings: *Characteristic* means that results are strictly related to a particular geometrical morphology; *Limit State Curve* represents the plot of the c-ϕ relation when in Bishop's formula is considered F=1 (which is the deterministic condition of imminent failure).

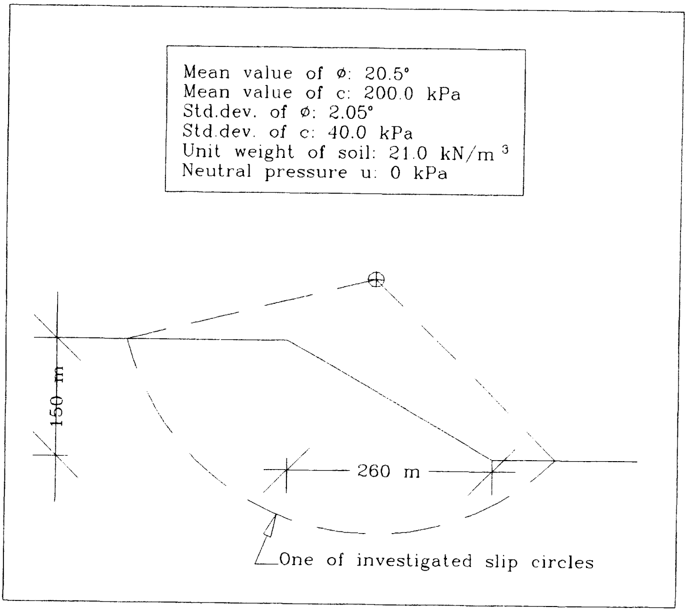

Figure 1. Slope geometry and soil mechanics parameters

In this way, sketching in c-ϕ plane the curve, it defines two regions: the "Safe Region" and the "Unsafe Region".

As a second step, using the same computer code, τ_m and σ_m have been obtained and by them the *Characteristic Resistance Envelope (CRE)* has been plotted.

The two curves allow linking quickly and easily the limit stress state to the mechanical properties.

The *CLSC* is used as a failure curve in Low's probability approach (Low[9]). Indeed, cohesion and friction angle has been considered random variables and represented in the random variables plane. Subsequently, onto the same plane Low's dispersion ellipse can be plotted.

It's easy to understand that if the ellipse 1-σ is all in the "Safe Region" the calculated β_{min} coefficient will be, certainly, greater then the unit. This coefficient itself, will be the measure of the reliability.

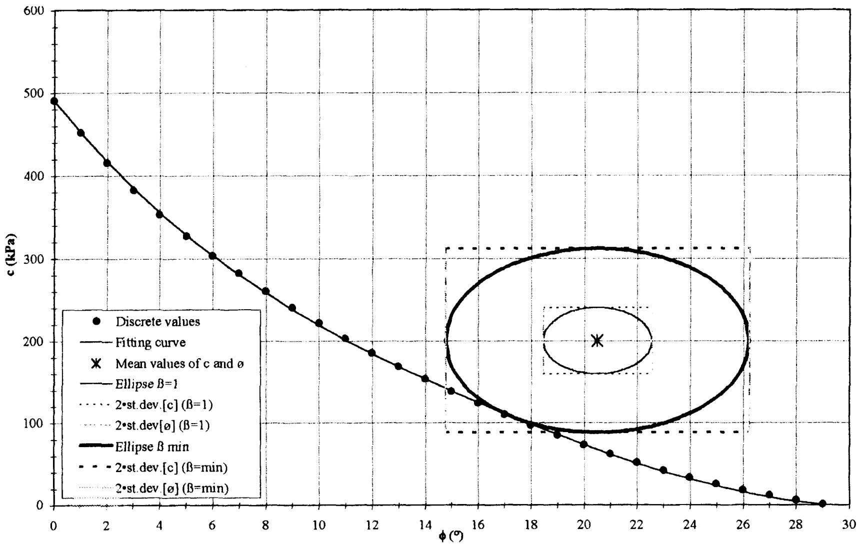

Figure 2. *Characteristic Limit State Curve* and dispersion ellipse
($r_{c,\phi} = 0.00$, β_{min}=2.794)

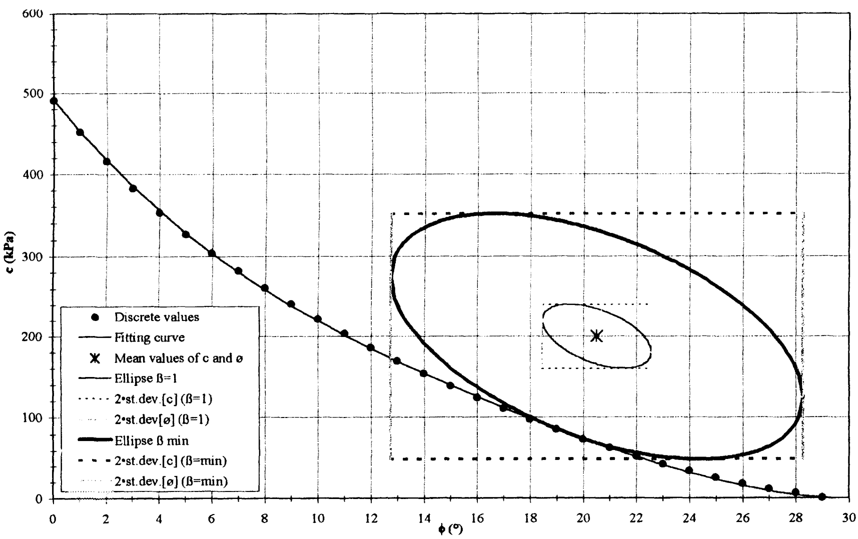

Figure 3. *Characteristic Limit State Curve* and dispersion ellipse
($r_{c,\phi} = -0.50$, β_{min}=3.788)

Then, considering the point that Shinozuka defined as the "design point" (the point in which the ellipse is tangent to the *CLSC*) by Mohr-Coulomb criterion the "failure envelope" can be constructed. It will be predictable that in stress plane "failure envelope" will be tangent to the characteristic resistance envelope (Figure 4). At the same time we can infer the value of "Safety Factor" comparing with Reliability Index value.

Table 1. Summary of data and results

Mechanical properties of soil	
Mean value of ϕ (°)	20.5
Mean value of c (kPa)	200.0
Standard deviation of ϕ (°)	2.05
Standard deviation of c (kPa)	40.00
Coefficient of correlation between c and ϕ: 0.00	
β_{min}	2.794
ϕ value for β_{min} (°)	17.3
c value for β_{min} (kPa)	107.3
ϕ value for slip circle nearest to that of β_{min} (°)	17.0
c value for slip circle nearest to that of β_{min} (kPa)	111.3
Δ_x of slip circle toe nearest to that of β_{min} from slope foot (m)	0.00
X of centre of slip circle nearest to that of β_{min} (m)	1040.0
Y of centre of slip circle nearest to that of β_{min} (m)	330.0
Coefficient of correlation between c and ϕ: −0.50	
β_{min}	3.788
ϕ value for β_{min} (°)	19.5
c value for β_{min} (kPa)	80.1
ϕ value for slip circle nearest to that of β_{min} (°)	19.0
c value for slip circle nearest to that of β_{min} (kPa)	85.7
Δ_x of slip circle toe nearest to that of β_{min} from slope foot (m)	0.00
X of centre of slip circle nearest to that of β_{min} (m)	1060.0
Y of centre of slip circle nearest to that of β_{min} (m)	360.0
Evaluation of Deterministic "Safety Factor"	
Deterministic "Safety Factor" F	1.471
σ value corresponding to F (kPa)	571.4
τ value on the "failure envelope" corresponding to F (kPa)	413.6
τ value on *CRE* corresponding to F (kPa)	281.3

Figures 2 and 3 shows that, for the same *CLSC* on c-ϕ plane, the change of correlation factor between cohesion and friction angle deals to different β_{min} values.

In the particular case of absence of correlation between these two parameters ($r_{c,\phi} = 0.00$), β_{min} is equal to 2.794 and the tangent point co-ordinates are c=107.3 kPa and ϕ=17.3°. On the other hand, for a negative correlation between the mentioned parameters ($r_{c,\phi} = -0.50$), β_{min} is equal to 3.788 and the tangent point co-ordinates are c=80.1 kPa and ϕ=19.5° (results in the two cases are summarised in Table 1).

The two mentioned examples show that, with negative correlation between cohesion and friction angle, the tangent point between ellipse and *CLSC* is on the right of that determined in the absence of correlation: it means that, at failure point, cohesion decreases and friction angle increases.

Mohr' s failure line is plotted for these values in Figure 4.

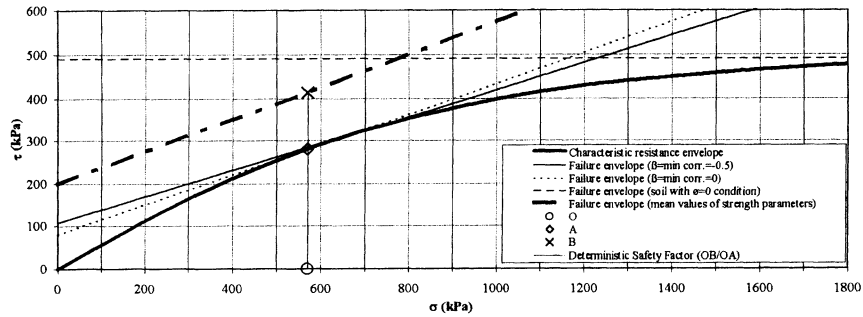

Figure 4. *Characteristic Resistance Envelope &* Mohr-Coulomb criterion Deterministic "Safety Factor" F=1.471)

In that figure is also reported "Safety Factor" calculation as defined by Casagrande[1] as the ratio between available shear strength and required shear strength. Its value is insensitive to coefficient of correlation so that it remains constant (F=1.471) without taking into any account that the β_{min} has changed.

So far, has been described the Overall Approach in a simple case but its synthetic point of view has a large applicability in practicable stability problems. Furthermore, developing computer code will make the Overall Approach much more flexible and allow us to analyse real problems.

6 Conclusions

The proposed procedure allows to make a comparison between the main two methods in geotechnical stability evaluations: deterministic and probability approach. As a matter of fact, Casagrande's deterministic concept of a *Characteristic Resistance Envelope* has been enriched by probabilistic implications. On the other hand, the probabilistic point of view illustrated, permit to point out the great influence of parameters such as correlation coefficient which is ignored by deterministic "Safety Factor".

References

[1] Casagrande A., Notes on the design of earth dams, *Journal of the Boston Society of Civil Engineers*, **37-4**, pp. 405-429, 1950.

[2] Cherubini C., Cucchiararo L., Giasi C.I., Valutazione della stabilità dei pendii in ottica probabilistica, *Proc. IV Convegno Internazionale di Geoingegneria "Difesa e valorizzazione del suolo e degli acquiferi"*, Torino, pp. 57-68, 1992.

[3] Cherubini C., La valutazione della sicurezza nella stabilità dei versanti, *Proc. Convegno Internazionale "La prevenzione delle catastrofi idrogeologiche: il contributo della ricerca scientifica"*, Alba, Italy, 1996.

[4] Cherubini C., Data and Considerations on the variability of geotechnical properties of soils, *Advanced in Safety and Reliability*, **2**, pp. 1583-1594, 1997.

[5] Federico A., Adamo S., Simeone V., Il metodo degli inviluppi di resistenza per l'analisi di stabilità dei pendii, *Geologia applicata e idrogeologia*, pp. 271-288, 1996.

[6] Hasofer M.A.,Lind C.N., Exact and invariant second-moment code format, *Proc. of the American Society of Civil Engineers*, **100-No.EM1**, pp. 111-121, 1974.

[7] Janbu N., *Stability analysis of slopes with dimensionless parameters*, Harvard Soil Mechanics Series, **46**, pp. 1-81, 1954.

[8] Low B.K, Tang H.W., Efficient reliability evaluation using spreadsheet, *Journal of Engineering Mechanics*, **123-7**, pp. 749-752, 1997.

[9] Low B.K., Reliability analysis of rock wedges, *Journal of Geotechnics and Environmental Engineering*, **123-6**, pp. 498-505, 1997.

[10] Sparks A.D.W., Aspects of slope stability and risk analysis, *Landslides. Glissements de Terrain, ISL 96*, **2**, pp. 1363-1368, 1996.

[11] Tabba M.M., Deterministic versus risk analysis of slope stability, *International Symposium on Landslides*, pp. 491-498, 1984.

[12] Varghese P.C., A new approach to the analysis of stability of slopes, *Journal of the Indian Roads Congress*, **19-1**, pp. 23-34, 1956.